

JOHNSON COUNTY COMMUNITY COLLEGE
MATH 172 COMPASS Trigonometry Review

Section I. Trigonometric Functions:

Find the exact value of each expression.

1. $\cos \frac{\pi}{3} + \sin \frac{\pi}{2}$

2. $4 \cos 60^\circ + 3 \tan \frac{\pi}{3}$

3. $3 \sin \frac{2\pi}{3} - 4 \cos \frac{5\pi}{2}$

4. $4 \csc \frac{3\pi}{4} - \cot \left(-\frac{\pi}{4} \right)$

5. $\cos \frac{\pi}{2} - \sec \left(-\frac{\pi}{2} \right)$

6. $\sin 270^\circ + \cos (-180^\circ)$

Find the exact value of each of the remaining trigonometric functions.

7. $\cos \theta = -\frac{3}{5}, \sin \theta < 0$

8. $\csc \theta = -\frac{5}{3}, \cot \theta < 0$

9. $\tan \theta = -\frac{2}{3}, 90^\circ < \theta < 180^\circ$

10. Find the height of a building if it casts a shadow 80 feet long when the angle of elevation of the sun is 25° .

11. A circle has a radius 16.4. Find the length of the arc cut by a central angle of $\frac{3\pi}{4}$. Round your answer to one decimal place.

Section II. Graphs of Trigonometric Functions:

1. Determine the amplitude of each function without graphing:

a. $y = \sin (2x)$

b. $y = 3 \cos (4x)$

c. $y = -2 \cos x$

2. Determine the period of each function without graphing:

a. $y = \sin (2x)$

b. $y = 3 \cos (4x - \pi)$

c. $y = -2 \cos x$

3. Find the phase shift of each function (indicate left or right):

a. $y = 2 \cos\left(\frac{1}{3}x\right)$

b. $y = -6 \sin(2\pi x - 2)$

4. Determine the vertical shift of each function:

a. $y = 1 + \sin(2x - \pi)$

b. $y = -2 - \cos\left(2x - \frac{\pi}{2}\right)$

5. Sketch one period of each function:

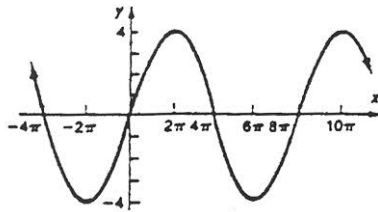
a. $y = \left(\frac{3}{2}\right) \cos(6x + 3\pi)$

b. $y = 2 \cos\left(\frac{1}{3}x\right)$

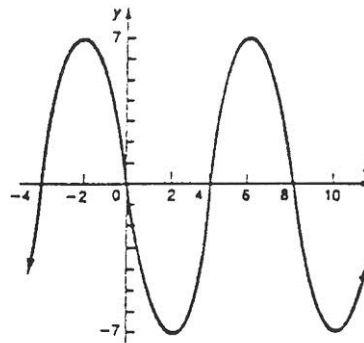
c. $y = -7 \sin\left(\frac{\pi}{3}x + \frac{4}{3}\right)$

6. Find a function whose graph is given below:

a.



b.

7. Graph the following on the interval $[-2\pi, 2\pi]$ using quadrantal angles. Give exact values for the y-coordinate of each point.

a. $f(x) = 3 \sin x + \cos 2x$

b. $f(x) = \sin x + \sin 2x$

Find the exact value of each expression:

8. $\cos^{-1} 0$

9. $\sin^{-1}\left(-\frac{1}{2}\right)$

10. $\tan^{-1}(-\sqrt{3})$

11. $\cos\left(\sin^{-1} \frac{3}{5}\right)$

$$12. \quad \csc \left(\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right)$$

13. Write an equivalent expression that involves x only. (Assume x is positive.)

$$\sin \left(\cos^{-1} \frac{1}{x} \right)$$

Section III. Analytic Trigonometry:

Prove each identity.

$$1. \quad \sin x \csc x - \sin^2 x = \cos^2 x$$

$$2. \quad (1 - \cos^2 x)(1 + \cot^2 x) = 1$$

$$3. \quad 4 \sin^2 x + 2 \cos^2 x = 4 - 2 \cos^2 x$$

$$4. \quad 1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$$

$$5. \quad (2 \sin 2\theta)(1 - 2 \sin^2 \theta) = \sin 4\theta$$

6. Find the exact value of the expression:

a. $\tan 105^\circ$

b. $\sin \left(\frac{5\pi}{8} \right)$

c. $\sin 70^\circ \cos 40^\circ - \cos 70^\circ \sin 40^\circ$

7. If $\cos A = \frac{4}{5}$ where $0 < A < \frac{\pi}{2}$ and $\cos B = \frac{5}{13}$ where $-\frac{\pi}{2} < B < 0$

a. Find the exact value of $\tan (A + B)$.

b. Find the exact value of $\cos 2B$.

c. Find the exact value of $\sin 2A$.

8. If $\tan A = -\frac{4}{3}$ where $\frac{\pi}{2} < A < \pi$ and $\cot B = \frac{12}{5}$ where $\pi < B < \frac{3\pi}{2}$

a. Find the exact value of $\sin (A - B)$.

b. Find the exact value of $\cos (A + B)$.

c. Find the exact value of $\tan 2B$.

9. If $\sin A = -\frac{5}{13}$ where $-\frac{\pi}{2} < A < 0$ and $\cos B = -\frac{4}{5}$ where $\frac{\pi}{2} < B < \pi$
- Find the exact value of $\sin\left(\frac{B}{2}\right)$.
 - Find the exact value of $\cos\left(\frac{A}{2}\right)$.

Solve each equation on the interval $0 \leq x < 2\pi$.

- $\sin x = -\frac{\sqrt{3}}{2}$
- $\tan x = -\sqrt{3}$
- $\cos 2x = 0$
- $\sin 3x = 1$
- $\tan x = 25$
- $\cos x = \sec x$
- $\cos 2x = \sin x$
- $\sin 2x - \sin x - 2 \cos x = -1$
- $2 \cos^2 x + \cos x - 1 = 0$

Section IV. Oblique Triangles and Polar Coordinates:

Find the remaining angle(s) and side(s) of each triangle, if it exists.

- $A = 50^\circ, B = 30^\circ, a = 1.0$
- $a = 3.0, c = 1.0, C = 110^\circ$
- $a = 3.0, A = 10^\circ, b = 4.0$
- $a = 2.0, b = 3.0, C = 60^\circ$

Find the area of each triangle.

- $a = 2.0, b = 3.0, C = 40^\circ$
- $a = 4.0, b = 2.0, c = 5.0$

7. The polar coordinates of a point are $\left(3, \frac{\pi}{6}\right)$. Find the exact values of the rectangular coordinates of the point.

Section V.

Write the complex number in trigonometric form.

1. $-1 - i$

Write the complex number in the standard form $a + bi$.

2. $2(\cos 150^\circ + i \sin 150^\circ)$

Find zw and $\frac{z}{w}$. Leave your answers in trigonometric form.

3. $z = \cos 80^\circ + i \sin 80^\circ$

$w = \cos 50^\circ + i \sin 50^\circ$

Write each expression in the standard form $a + bi$.

4. $[3(\cos 20^\circ + i \sin 20^\circ)]^3$

5. $(1 - \sqrt{3}i)^6$

Use the vectors $v = -2i + j$ and $w = 4i - 3j$ for 6 and 7.

6. $|v| =$

7. $|w| =$

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ANSWER KEY
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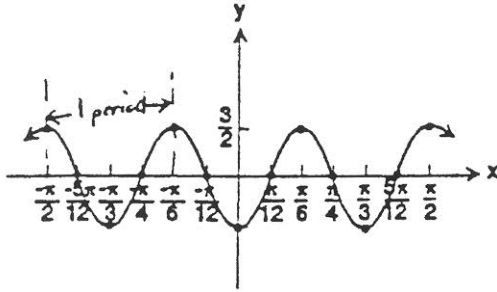
Section I.

1. $\frac{3}{2}$
2. $2 + 3\sqrt{3}$
3. $\frac{3\sqrt{3}}{2}$
4. $4\sqrt{2} + 1$
5. undefined
6. -2
7. $\sin \theta = -\frac{4}{5}, \tan \theta = \frac{4}{3}, \csc \theta = -\frac{5}{4}, \sec \theta = -\frac{5}{3}, \cot \theta = \frac{3}{4}$
8. $\sin \theta = -\frac{3}{5}, \cos \theta = \frac{4}{5}, \tan \theta = -\frac{3}{4}, \sec \theta = \frac{5}{4}, \cot \theta = -\frac{4}{3}$
9. $\sin \theta = \frac{2}{\sqrt{13}}, \cos \theta = \frac{-3}{\sqrt{13}}, \csc \theta = \frac{\sqrt{13}}{2}, \sec \theta = -\frac{\sqrt{13}}{3}, \cot \theta = -\frac{3}{2}$
10. $h \approx 37$ feet
11. ≈ 38.6

Section II.

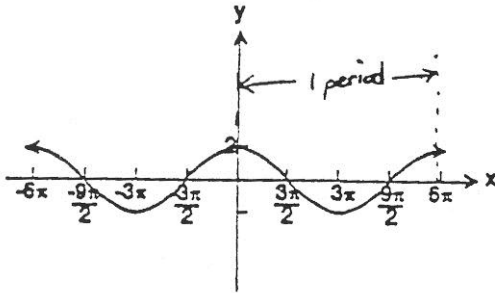
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|-------------|-----------------------------|-----------|
| 1. a. 1 | b. 3 | c. 2 |
| 2. a. π | b. $\frac{\pi}{2}$ | c. 2π |
| 3. a. none | b. $\frac{1}{\pi}$ to right | |
| 4. a. 1 | b. -2 | |

5. a. Amplitude = $\frac{3}{2}$, Period = $\frac{2\pi}{6} = \frac{\pi}{3}$, Phase Shift = $\frac{\pi}{2}$ to left



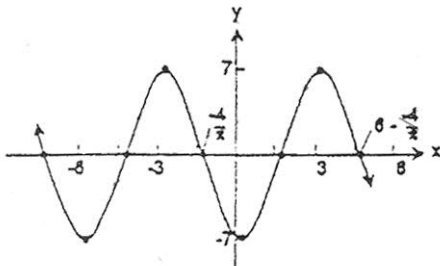
$$y = \frac{3}{2} \cos(6x + 3\pi)$$

- b. Amplitude = 2, Period = $\frac{2\pi}{\frac{1}{3}} = 6\pi$



$$y = 2 \cos \frac{1}{3}x$$

- c. Amplitude = 7 ($A = -7 < 0$), Period = $\frac{2\pi}{\frac{\pi}{3}} = 6$, Phase Shift = $\frac{4}{\pi}$ to left.

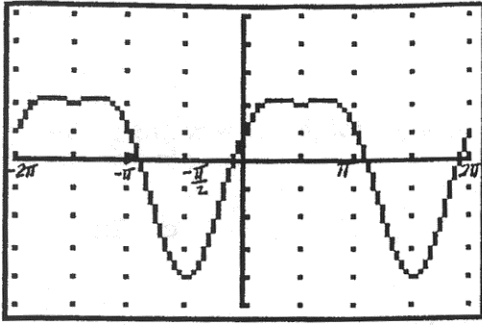


$$y = -7 \sin \left(\frac{\pi}{3}x + \frac{4}{3} \right)$$

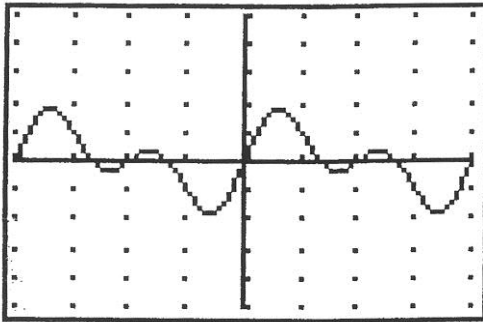
6. a. $y = 4 \sin \left(\frac{x}{4} \right)$

b. $y = -7 \sin \left(\frac{\pi}{4} \right) x$

7. a.

 $(0, 1)$ $\left(\frac{\pi}{2}, 2\right) \quad \left(-\frac{\pi}{2}, -4\right)$ $(\pi, 1) \quad (-\pi, 1)$ $\left(\frac{3\pi}{2}, -4\right) \quad \left(-\frac{3\pi}{2}, 2\right)$ $(2\pi, 1) \quad (-2\pi, 1)$

7. b.

 $(0, 0)$ $\left(\frac{\pi}{2}, 1\right) \quad \left(-\frac{\pi}{2}, -1\right)$ $(\pi, 0) \quad (-\pi, 0)$ $\left(\frac{3\pi}{2}, -1\right) \quad \left(-\frac{3\pi}{2}, 1\right)$ $(2\pi, 0) \quad (-2\pi, 0)$

8. $\frac{\pi}{2}$

9. $\frac{-\pi}{6}$

10. $\frac{-\pi}{3}$

11. $\frac{4}{5}$

12. $\frac{-2}{\sqrt{3}}$

13. $\frac{\sqrt{x^2 - 1}}{x}$

Section III.

$$6. \quad \text{a. } \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \quad \text{b. } \frac{\sqrt{2 + \sqrt{2}}}{2} \quad \text{c. } \frac{1}{2}$$

$$7. \quad \text{a. } \tan(A + B) = \frac{-33}{56} \quad \text{b. } \cos 2B = \frac{-119}{169} \quad \text{c. } \sin 2A = \frac{24}{25}$$

$$8. \quad \text{a. } \sin(A - B) = \frac{-63}{65} \quad \text{b. } \cos(A + B) = \frac{56}{65} \quad \text{c. } \tan 2B = \frac{120}{119}$$

$$9. \quad \text{a. } \sin\left(\frac{B}{2}\right) = \frac{3\sqrt{10}}{10} \text{ or } \frac{3}{\sqrt{10}} \quad \text{b. } \cos\left(\frac{A}{2}\right) = \frac{5\sqrt{26}}{26} \text{ or } \frac{5}{\sqrt{26}}$$

$$10. \quad \frac{4\pi}{3} \text{ or } \frac{5\pi}{3}$$

$$11. \quad \frac{2\pi}{3} \text{ or } \frac{5\pi}{3}$$

$$12. \quad \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$13. \quad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$14. \quad 4.6724 \text{ or } 1.5308$$

$$15. \quad 0 \text{ or } \pi$$

$$16. \quad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$17. \quad \frac{\pi}{2} \text{ or } \frac{\pi}{3}, \frac{5\pi}{3}$$

$$18. \quad \frac{\pi}{3}, \frac{5\pi}{3}, \text{ or } \pi$$

Section IV.

1. $C = 100^\circ$, $b = 0.65$, $c = 1.3$
2. No triangle
3. Two triangles: $B_1 = 13^\circ$, $C_1 = 157^\circ$, $c_1 = 6.8$; $B_2 = 167^\circ$, $C_2 = 3^\circ$, $c_2 = 0.90$
4. $c = 2.6$, $A \approx 41^\circ$, $B \approx 79^\circ$
5. 1.9
6. 3.8
7. $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

Section V.

1. $\sqrt{2} (\cos 225^\circ + i \sin 225^\circ)$
2. $-\sqrt{3} + i$
3. $zw = \cos 130^\circ + i \sin 130^\circ$; $\frac{z}{w} = \cos 30^\circ + i \sin 30^\circ$
4. $\left(\frac{27}{2}\right)(1 + \sqrt{3}i)$
5. 64
6. $\sqrt{5}$
7. 5