Advanced Algebra Topics COMPASS Review – revised Summer 2013

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1. If $f(x)=2x^3-2x+\frac{6}{x}$ and $g(x)=(x+1)(x-3)$, find $f(2)+g(-3)$.
   a. 12 b. 63 c. 27 d. 0

2. If $f(x)=x^2-3x$, $g(x)=2x-1$, and $h(x)=2x^2+5$, find $f(x)-[g(x)\cdot h(x)]$.
   a. $-x^2-5x-4$ b. $-4x^3+3x^2-13x+5$ c. $4x^3-x^2+7x-5$ d. $-2x^4-5x^2-4$

3. If $f(x)=3x-2$ and $g(x)=12x^3-8x^2+9x-6$, find $\frac{g(x)}{f(x)}$.
   a. $\frac{3x-2}{12x^3-8x^2+9x-6}$ b. $4x^2+3-\frac{12}{3x-2}$ c. $-4x^2+9x-3$ d. $4x^2+3$

4. If $f(x)=3x-2$ and $g(x)=x^2-4x+1$, find $(g\circ f)(x)$.
   a. x^2-7x+3 b. x^2-4x+1 c. $9x^2-24x+13$ d. $3x^2-12x+1$

5. If $f(x)=4x^2-2x+1$, find $f(x+3)$.
   a. $4x^2+22x+31$ b. $4x^2-2x+4$ c. $4x^2-2x+31$ d. $4x^2+10x+7$

6. If $f(x)=x(3x-2)$, find $f(x+h)-f(x)$.
   a. $6xh+3h^2-2h$ b. h c. $3x^2h^2+3h^2-2h$ d. $6x-2$

7. If $f(x)=3x-2$ and $g(x)=x^2-4x+1$, find $f[g(2)]$.
   a. -12 b. -1 c. 1 d. -11

8. If $f(x)=\sqrt[3]{4x+1}$, find $f^{-1}(x)$.
   a. $f^{-1}(x)=\frac{x^3-1}{4}$ b. $f^{-1}(x)=(-4x-1)^3$ c. $f^{-1}(x)=\sqrt[3]{\frac{1}{4}x+1}$ d. $f^{-1}(x)=\frac{1}{\sqrt[3]{4x+1}}$

9. If $f(x)=6x-3$, and g is a function such that $f(g(x))=g(f(x))=x$ for all values of x, find $g(x)$.
   a. $g(x)=3x-6$ b. $g(x)=\frac{1}{6}x+\frac{1}{2}$ c. $g(x)=36x-21$ d. $g(x)=6x-3$

10. If $f(x)$ contains the point (4,1), then $f^{-1}(x)$ must contain what point?
   a. $(-4,1)$ b. $(1,4)$ c. $(4,-1)$ d. $(1,\frac{1}{4})$

11. The value V in dollars of a piece of equipment t months after its purchase is given by the function $V(t)=7250-125t$. Find $V^{-1}(50)$.
   a. 7247.5 b. 0.001 c. 1000 d. 57.6

12. Find the domain of $f(x)=\dfrac{x^2-1}{x^2-4}$.

a. $(-\infty,0)\cup(0,\infty)$
c. $(-\infty,-2)\cup(-2,-1)\cup(1,2)\cup(2,\infty)$
b. $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
d. $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Which describes all real values of x for which $f(x)=\sqrt{3x+9}-1$ is a real number?

a. $\{x \mid x\ge -3\}$
b. $\{x \mid x>-3\}$
c. $\{x \mid x>0\}$
d. $\{x \mid 0\le x\le 3\}$

14. If the minimum value of $f(x)$ occurs at $x=3$, at which value of x does the minimum value of $f(x-2)+1$ occur?

a. $x=2$
b. $x=5$
c. $x=4$
d. $x=6$

15. Which describes the set of all real values of y for which $y=f(x)$ if $f(x)=\dfrac{x^2-1}{x^2-4}$?

a. $\{y \mid y\ne -2 \text{ and } y\ne 2\}$
b. $\{y \mid y\ge -1\}$
c. $\{y \mid y>1 \text{ or } y\le \frac{1}{4}\}$
d. all real numbers

16. Find the range of $f(x)=\sqrt{3x+9}-1$.

a. $[-3,\infty)$
b. $[-1,\infty)$
c. $[0,\infty)$
d. [0,3]

17. If $f(x)=kx^3-2x$ and $f(-2)=6$, find $f(4)$.

a. -168
b. -24
c. 216
d. -56

18. If $x=4y+1$ and $z=3x-5$, find an expression for z in terms of y.

a. $z = 4y + 1$
b. $z = 12y - 2$
c. $z = 12y - 19$
d. $z = 3y - 5$

19. The demand d for items priced at p dollars per item is given by $d(p)=\dfrac{9000}{p+1}-3$, and the revenue R of selling d items at price p is given by $R=pd$. Find R as a function of p.

a. $R(p)=\dfrac{9000}{p+1}-3$
b. $R(p)=\dfrac{-3p+8997}{p^2+p}$
c. $R(p)=\dfrac{-3p^2+8997p}{p+1}$
d. $R(p)=\dfrac{9000}{p+1}-3p$

20. Find the vertex of the parabola $f(x)=3x^2+6x-5$.

a. $(-3,-5)$
b. $(1,-8)$
c. $(-1,-2)$
d. $(-1,-8)$

21. Find the solutions to the equation $8x-2x^2=3$

a. $x=2\pm 2\sqrt{10}$
b. $x=8\pm\dfrac{\sqrt{10}}{2}$
c. $x=8\pm 2\sqrt{10}$
d. $x=2\pm\dfrac{\sqrt{10}}{2}$

22. Which equation has $\{0,2,-4\}$ as its solution set?

a. $2x-4=0$
b. $x(x-2)(x+4)=0$
c. $x(x+2)(x-4)=0$
d. $x(2x-4)=0$

23. What are the zeros of the function $f(x)=x^3+2x^2-9x-18$?

a. $x = 3, x = -3, x = 2$
c. $x = 3, x = -3, x = -2$
b. $x = 0, x = -3, x = 3, x = -2, x = 2$
d. $x = 0, x = 9, x = 2, x = -2$

24. Which cubic function has $x=0, x=-2,$ and $x=5$ as zeros?

a. $f(x)=x^3-7x^2+10x$ b. $f(x)=x^3+7x^2+10x$ c. $f(x)=x^3+3x^2-10x$ d. $f(x)=x^3-3x^2-10x$

25. If the domain of $f(x)=\left(x^2-9\right)\left(x^2+9\right)$ is all real numbers, what is the solution set for $f(x)=0$?

a. $\{9,-9\}$ b. $\{3,-3\}$ c. $\{3,-3,-9\}$ d. $\{3,-3,3i,-3i\}$

26. Factor $f(x)=x^4-81$ completely over the complex numbers.

a. $f(x)=(x-3)(x+3)(x-3)(x+3)$
c. $f(x)=(x+3)(x-3)(x+3i)(x-3i)$
b. $f(x)=(x+3)(x-3)(x^2+9)$
d. $f(x)=(x+3i)(x-3i)(x^2-9)$

27. Which quadratic function has a corresponding graph with a vertex at $(2,-3)$ and contains the point $(-2,5)$?

a. $f(x)=(x-2)^2-3$ b. $f(x)=\frac{1}{2}x^2-2x-1$ c. $f(x)=(x+2)^2-3$ d. $f(x)=x^2-4x-1$

28. Simplify $(a^2)^{\frac{2}{3}}$

a. $a^{\frac{8}{3}}$ b. $a^{\frac{4}{3}}$ c. $a^{\frac{2}{3}}$ d. $a^{\frac{1}{3}}$

29. Simplify $3a^{\frac{1}{2}}b^{\frac{3}{2}}\cdot 2a^{\frac{3}{2}}b^{\frac{5}{2}}$

a. $6a^{\frac{3}{4}}b^{\frac{15}{4}}$ b. $5a^2b^4$ c. $6a^3b^8$ d. $6a^2b^4$

30. Simplify $\left(\dfrac{2x^2y^{-5}}{3x^{-3}y^6}\right)^{-3}$

a. $\dfrac{2y^3}{3x^3}$ b. $\dfrac{8y^3}{27x^3}$ c. $\dfrac{27y^{33}}{8x^{15}}$ d. $\dfrac{3x^{15}}{2y^{33}}$

31. Simplify, assuming that the variables represent positive numbers: $\sqrt{8a}\cdot\sqrt[3]{4a^2}$

a. $4a\sqrt{2a}$ b. $2a\sqrt[3]{2}$ c. $4a\sqrt[6]{2a}$ d. Not possible

32. If $25^{2x+1}=\dfrac{1}{5^{3x-4}}$ then $x=$?

a. $-\frac{7}{5}$ b. -6 c. $\frac{2}{7}$ d. 5

33. If $\dfrac{x^{a^2}}{x^{5a}}=x^{a-5}$ for all real values of x such that $x\neq 0$, find the value(s) of a.

a. $a=5$ b. $a=0$ c. $a=\pm 5$ d. $a=5, a=1$

34. Rewrite using logarithmic notation: $M^x = y$

a. $\log_x y = M$ b. $\log_y M = x$ c. $\log_M x = y$ d. $\log_M y = x$

35. If $\log_2\left(\frac{1}{4}\right) = x$, then $x = ?$

a. $\frac{1}{2}$ b. 2 c. -2 d. $\frac{1}{16}$

36. Express as a single logarithm: $2\log_{10} 3 + 4\log_{10} y - 6\log_{10} z - 8\log_{10} t$

a. $-6\log_{10}(3yzt)$ b. $\log_{10}(6+4y-6z-8t)$ c. $\log_{10}\left(\dfrac{9y^4}{z^6t^8}\right)$ d. $-8\log_{10}(3+y-z-t)$

37. If $10^{2x-1} - 50 = 50$, find the value of x.

a. $\frac{3}{2}$ b. 0 c. $\frac{1}{2}$ d. 3

38. If $\ln(3x-4) = 0$, find the value of x.

a. $\frac{5}{3}$ b. $\frac{4}{3}$ c. $\frac{e+4}{3}$ d. 1

39. The function $T(t) = 70 + 28.6e^{-0.14t}$ gives the temperature of an object in a room t hours after midnight. Find t when $T = 85$. Give your answer rounded to the nearest tenth.

a. 1.1 b. 0.2 c. 70.0 d. 4.6

40. Simplify: $7i^{23} - 2i(3-4i) - (1+2i)^2$.

a. $3-3i$ b. $-5-17i$ c. $-5-13i$ d. $3-i$

41. The product of two complex numbers is $2+3i$. One of the complex numbers is $-1+4i$. Find the other complex number.

a. $-14+5i$ b. $\frac{10}{17} - \frac{11}{17}i$ c. $\frac{10}{13} + \frac{11}{13}i$ d. $3-i$

42. If z is a complex number with $\bar{z}$ its complex conjugate, and $\bar{z} - 2z = 6i$, find z.

a. -6 b. $\frac{1}{2} - 3i$ c. $-2i$ d. $2i$

43. If the nth term of a sequence is given by $(-1)^{n+1}\dfrac{n+1}{n^2}$, find the $(n+1)$th term.

a. $(-1)^{n+1}\dfrac{n+1}{n^2}$ b. $\dfrac{n+1}{n^2}$ c. $(-1)^{n+2}\dfrac{n+2}{n^2+2n+1}$ d. $(-1)^{n+2}\dfrac{n+2}{n^2+1}$

44. A certain arithmetic sequence has $a_1 = 56$ and $a_{11} = 26$. Find a_{17}.

a. 21 b. 18 c. 3 d. 8

45. If $2, x, 7$ are the first three terms of a geometric sequence of positive real numbers, find the common ratio r.

a. $\frac{7}{2}$ b. $\pm\sqrt{\frac{7}{2}}$ c. $\frac{5}{2}$ d. $\sqrt{\frac{7}{2}}$

46. Which of the following could be the seventh term in a geometric sequence with $a_2 = 3$ and $a_4 = 9$?

a. $\sqrt{3}$ b. 2187 c. $27\sqrt{3}$ d. 729

47. A recursive sequence is given by $a_1 = 2+3i$, and $a_n = i \cdot a_{n-1}$ for $n \geq 2$, where $i = \sqrt{-1}$. Find a_5.

a. $3+2i$ b. $2+3i$ c. $-2-3i$ d. $32+243i$

48. Evaluate $\sum_{n=1}^{23}(6n-3)$.

a. 1587 b. 60 c. 1518 d. 135

49. A stack of cans has 60 cans in the bottom row, and each row above the bottom row has two less cans than the row below it. Find the total number of cans in the first 10 rows, starting with the bottom row.

a. 600 b. 580 c. 560 d. 510

50. Evaluate $\dfrac{9!}{3!\,6!}$

a. 1 b. 84 c. $\frac{1}{2}$ d. $\frac{45}{126}$

51. If $A = \begin{bmatrix} 8 & -2 \\ 3 & 7 \\ -5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 4 \\ 0 & 3 \\ -2 & 6 \end{bmatrix}$, find the entry in the first row and the first column for $A+5B$.

a. 11 b. 5 c. -7 d. 47

52. If $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} k & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 15 & 17 \end{bmatrix}$, then k =?

a. 3 b. 4 c. 8 d. $-\frac{3}{2}$

53. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ and $\begin{vmatrix} k & 1 \\ 4 & 2 \end{vmatrix}$ =10, then k = ?

a. $\frac{5}{4}$ b. 3 c. 10 d. 7

54. Give the z-value of the solution to the following system:

$$\begin{aligned} 4x - 5y &= 11 \\ 2x + z &= 7 \\ 2y + z &= 1 \end{aligned}$$

a. 2 b. -1 c. 1 d. 4

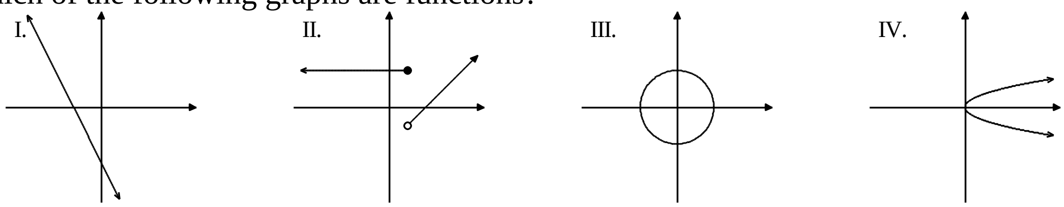
55. Which of the following is an augmented matrix for the given system? $\begin{aligned} 4x - 5y &= 11 \\ y - 2x &= 7 \end{aligned}$

a. $\left[\begin{array}{cc|c} 4 & -5 & 11 \\ 1 & -2 & 7 \end{array}\right]$ b. $\left[\begin{array}{cc|c} 4 & -5 & 11 \\ -2 & 1 & 7 \end{array}\right]$ c. $\left[\begin{array}{c|c} 4 & -2 \\ -5 & 1 \\ 11 & 7 \end{array}\right]$ d. $\left[\begin{array}{cc|c} 11 & 4 & -5 \\ 7 & 1 & -2 \end{array}\right]$

56. Which of the following gives the solution to $\dfrac{x^2 - 4}{-3} \geq x$?

a. $\varnothing$ b. $[1,3]$ c. $[-4,1]$ d. $(-\infty,-4] \cup [1,\infty)$

57. Which of the following graphs are functions?



a. I only b. I and II only c. I, III, and IV only d. all of I, II, III and IV

58. For the vectors $\mathbf{a} = \langle 1,3 \rangle$ and $\mathbf{b} = \langle 2,-5 \rangle$, find $\mathbf{a} - 3\mathbf{b}$.

a. $\langle -5,18 \rangle$ b. 13 c. -13 d. -11

59. For the vectors $\mathbf{a} = \langle 1,3 \rangle$ and $\mathbf{b} = \langle 2,-5 \rangle$, find $\mathbf{a} \cdot \mathbf{b}$.

a. $\langle -5,18 \rangle$ b. 13 c. -13 d. -11

60. If the point $(2,-4)$ is reflected across the *x*-axis, then translated by the vector $\langle 3,-1 \rangle$, what will be the coordinates of the resulting point?

a. $(5,-5)$ b. $(1,3)$ c. $(1,-5)$ d. $(5,3)$

61. Given sets $A = \{1,3,7\}$ and $B = \{2,8,20\}$, which of the following is a function from *A* to *B*?

a. $f(x) = -3x + 23$ b. $f(x) = 6x + 2$ c. $f(x) = x^2 - x + 2$ d. $f(x) = 3x - 1$

62. Data is collected on two variables, *x* and *y*, shown in the table below. Which of the following equations describes the relationship between *x* and *y*?

x	*y*
20	6
40	16
80	26

a. $y = x - 14$ b. $y = 10\log_2\left(\frac{x}{10}\right) - 4$ c. $y = 2x + 10$ d. $y = \frac{1}{10}\log\left(\frac{x}{10}\right) + 6$

Answers to Advanced Algebra Topics COMPASS Review

1. c	17. b	33. d	48. a
2. b	18. b	34. d	49. d
3. d	19. c	35. c	50. b
4. c	20. d	36. c	51. c
5. a	21. d	37. a	52. a
6. a	22. b	38. a	53. d
7. d	23. c	39. d	54. b
8. a	24. d	40. b	55. b
9. b	25. b	41. b	56. c
10. b	26. c	42. c	57. b
11. d	27. b	43. c	58. a
12. d	28. b	44. d	59. c
13. a	29. d	45. d	60. d
14. b	30. c	46. c	61. d
15. c	31. c	47. b	62. b
16. b	32. c		

Solutions to Advanced Algebra Topics COMPASS Review

1. $f(2) = 2(2)^3 - 2(2) + \frac{6}{2} = 16 - 4 + 3 = 15$, and $g(-3) = (-3+1)(-3-3) = (-2)(-6) = 12$, so $f(2) + g(-3) = 15 + 12 = 27$

2. $f(x) - \left[g(x) \cdot h(x)\right] = (x^2 - 3x) - (2x-1)(2x^2+5) = (x^2 - 3x) - (4x^3 - 2x^2 + 10x - 5) = x^2 - 3x - 4x^3 + 2x^2 - 10x + 5 = -4x^3 + 3x^2 - 13x + 5$

3. $\dfrac{g(x)}{f(x)} = \dfrac{12x^3 - 8x^2 + 9x - 6}{3x-2}$, then use long division to simplify:

$$\begin{array}{r|l} & 4x^2 \qquad\quad +3 \\ \hline 3x-2 & 12x^3 - 8x^2 + 9x - 6 \\ & \underline{-(12x^3 - 8x^2)} \\ & \qquad\qquad 9x - 6 \\ & \qquad\quad \underline{-(9x-6)} \\ & \qquad\qquad\qquad 0 \end{array}$$

Quotient is $4x^2 + 3$

4. $(g \circ f)(x) = g(f(x)) = g(3x-2) = (3x-2)^2 - 4(3x-2) + 1 = (9x^2 - 12x + 4) - 12x + 8 + 1 = 9x^2 - 24x + 13$

5. $f(x+3) = 4(x+3)^2 - 2(x+3) + 1 = 4(x^2 + 6x + 9) - 2(x+3) + 1 = 4x^2 + 24x + 36 - 2x - 6 + 1 = 4x^2 + 22x + 31$

6. $f(x+h) - f(x) = (x+h)(3(x+h) - 2) - (x(3x-2)) = (x+h)(3x + 3h - 2) - (3x^2 - 2x) = 3x^2 + 3xh - 2x + 3xh + 3h^2 - 2h - 3x^2 + 2x = 6xh + 3h^2 - 2h$

7. $g(2)=2^2-4(2)+1 \;=\; 4-8+1 \;=\; -3$, so $f[g(2)] \;=\; f(-3) \;=\; 3(-3)-2 \;=\; -9-2 \;=\; -11$

8. $y=\sqrt[3]{4x+1}$ Reverse the roles of the *x* and *y* to form the inverse relation.
$x=\sqrt[3]{4y+1}$ This is the inverse relation. Now solve for *y* to express as a function.
$x^3=\left(\sqrt[3]{4y+1}\right)^3 \quad\Rightarrow\quad x^3=4y+1 \quad\Rightarrow\quad \dfrac{x^3-1}{4}=y \quad\Rightarrow\quad f^{-1}(x)=\dfrac{x^3-1}{4}$

9. $g(x)$ is the inverse function for $f(x)$, so follow the same procedure as in question 8.
$y=6x-3$ Now reverse the roles of the *x* and *y* to form the inverse relation.
$x=6y-3$ This is the inverse relation. Now solve for *y* to express as a function.
$\dfrac{x+3}{6}=y$ can also be written as $\frac{1}{6}x+\frac{1}{2}=y$, so $g(x)=f^{-1}(x)=\frac{1}{6}x+\frac{1}{2}$

10. The *x* and *y* values swap, so an ordered pair in $f^{-1}(x)$ is (1,4).

11. The input and output values swap, so $V^{-1}(50)$ will be the value of the input that will have an output of 50 for the function $V(t)$.
$50=7250-125t \quad\Rightarrow\quad t=\dfrac{50-7250}{-125}=57.6$ months

12. The domain is all real values of *x*, except those that would make the denominator equal zero.
$x^2-4\neq 0 \quad\Rightarrow\quad (x-2)(x+2)\neq 0 \quad\Rightarrow\quad x\neq 2,\;\; x\neq -2$, so the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

13. $3x+9$ must be greater than or equal to zero in order to get real-valued outputs.
$3x+9\geq 0 \quad\Rightarrow\quad x\geq -3$, so the domain is $[-3,\infty)$, or using set notation, $\{x \mid x\geq -3\}$.

14. The graph of $f(x-2)+1$ will be the graph of $f(x)$, but shifted right 2 units and down 1 unit. Therefore, the *x*-coordinate of the minimum will be shifted right 2 units, so it will occur at $x=5$.

15. One way to answer this is to examine the graph to determine the range of $f(x)$. The graph has a horizontal asymptote at $y=1$, and when $x>2$ or $x<-2$, the graph is above the horizontal asymptote, so $y>1$. When x is between -2 and 2, the highest output occurs at $x=0$, with $y=f(0)=\frac{1}{4}$. Therefore the range, or set of real values of *y*, is $\{y \mid y>1 \text{ or } y\leq \frac{1}{4}\}$.

16. The outputs of the radical expression $\sqrt{3x+9}$ will be 0 or larger, and then the -1 term will decrease these values by 1. Therefore the range is $y\geq -1$, or in interval notation, $[-1,\infty)$.

17. Use $f(-2)=6$ to find k: $6=k(-2)^3-2(-2) \quad\Rightarrow\quad 6=-8k+4 \quad\Rightarrow\quad k=-\frac{1}{4}$
Then $f(4)=-\frac{1}{4}(4)^3-2(4) \;=\; -16-8 \;=\; -24$

18. Substitute $4y+1$ in place of *x* in the equation $z=3x-5$, then simplify: $z=3(4y+1)-5 \quad\Rightarrow\quad z=12y-2$

19. Substitute $\dfrac{9000}{p+1}-3$ in place of d in the equation $R=pd$, then simplify:

$$R=p\left(\frac{9000}{p+1}-3\right) = \frac{9000p}{p+1}-3p = \frac{9000p}{p+1}-\frac{3p(p+1)}{p+1} = \frac{9000p-3p^2-3p}{p+1} = \frac{-3p^2+8997p}{p+1}$$

20. Method 1: Complete the square to obtain the form $f(x)=a(x-h)^2+k$, where the vertex is (h,k):

$$f(x)=3x^2+6x-5 = 3(x^2+2x \quad)-5 = 3(x^2+2x+1)-5-3(1) = 3(x+1)^2-8$$

The vertex is $(-1,-8)$

Method 2: Use $x=\dfrac{-b}{2a}$ to find the x-coordinate, then substitute into $f(x)$ to find the y-coordinate:

$x=\dfrac{-6}{2(3)}=-1$, and $y=f(-1)=3(-1)^2+6(-1)-5 = -8$. The vertex is $(-1,-8)$.

21. Since the equation is quadratic, write it in standard form and use the quadratic formula:

$0=2x^2-8x+3$, and $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a} \quad\Rightarrow\quad x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(2)(3)}}{2(2)}=\dfrac{8\pm\sqrt{40}}{4}=\dfrac{8\pm2\sqrt{10}}{4}$

simplifies to $x=\dfrac{4\pm\sqrt{10}}{2}$ or $x=2\pm\frac{1}{2}\sqrt{10}$

22. Many equations could have this solution set, but the choices given are all polynomials in factored form, set equal to zero. This allows us to use the fact that if $x=a$ is a zero of a polynomial function, then $(x-a)$ is a factor of the polynomial. This allows us to write the equation $x(x-2)(x+4)=0$.

23. This function is factorable by grouping: $f(x)=x^2(x+2)-9(x+2)=(x^2-9)(x+2)=(x-3)(x+3)(x+2)$. Set $f(x)=0$ to get zeros of 3, – 3, and – 2.

24. The zeros will create factors of $(x)(x+2)(x-5)$. Multiply these out to get $f(x)=x^3-3x^2-10x$.

25. Set $0=(x^2-9)(x^2+9)$ and solve to get $x^2-9=0$ or $x^2+9=0$. The solutions to $x^2+9=0$ are nonreal, and since the domain is specified as real numbers, we discard the nonreal solutions. This means we have only $x^2-9=0 \Rightarrow (x+3)(x-3)=0 \Rightarrow x=\pm3$.

26. Factoring as a difference of squares we get $f(x)=(x^2-9)(x^2+9)$. The zeroes of x^2-9 are $x=\pm3$, as in question 25, and the zeroes of x^2+9 satisfy $x^2+9=0 \Rightarrow x^2=-9 \Rightarrow x=\pm\sqrt{-9}=\pm3i$. Using the fact that if $x=a$ is a zero of a polynomial function, then $(x-a)$ is a factor of the polynomial, we have the factored polynomial $f(x)=(x+3)(x-3)(x+3i)(x-3i)$.

27. Use the form of a parabola: $f(x)=a(x-h)^2+k$.

Insert the coordinates of the vertex for h and k: $f(x)=a(x-2)^2+(-3)$

Now use the other ordered pair to find a: $5=a(-2-2)^2+(-3) \quad \Rightarrow \quad \frac{1}{2}=a$

Now create the function using the a value: $f(x)=\frac{1}{2}(x-2)^2+(-3)$ or $f(x)=\frac{1}{2}x^2-2x-1$

28. Multiply the exponents together to get $a^{\frac{4}{3}}$.

29. Multiply the coefficients, and add the exponents on the like variables: $6a^{\frac{1}{2}+\frac{3}{2}}b^{\frac{3}{2}+\frac{5}{2}} \quad = \quad 6a^2b^4$

30. Simplify the outside power first, then the negative exponents, and then combine the like variables.

$$\frac{2^{-3}x^{-6}y^{15}}{3^{-3}x^9y^{-18}} \quad = \quad \frac{3^3y^{15}y^{18}}{2^3x^6x^9} \quad = \quad \frac{27y^{33}}{8x^{15}}$$

31. Write the radicals as fractional exponents, then use properties of exponents to simplify.

$$\sqrt{8a}\cdot\sqrt[3]{4a^2}=\left(2^3a\right)^{\frac{1}{2}}\left(2^2a^2\right)^{\frac{1}{3}} \;=\; 2^{\frac{3}{2}}a^{\frac{1}{2}}\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}} \;=\; 2^{\frac{13}{6}}a^{\frac{7}{6}} \;=\sqrt[6]{2^{13}a^7} \;=\; 2^2a^1\sqrt[6]{2^1a^1} \;=\; 4a\sqrt[6]{2a}$$

32. Start by writing both sides of the equation as powers of 5, then solve for x.

$$\left(5^2\right)^{2x+1}=5^{-(3x-4)} \quad \Rightarrow \quad 5^{4x+2}=5^{-3x+4} \quad \Rightarrow \quad 4x+2=-3x+4 \quad \Rightarrow \quad 7x=2 \quad \Rightarrow \quad x=\frac{2}{7}$$

33. Start by simplifying both sides as powers of x, then solve for a.

$$x^{a^2-5a}=x^{a-5} \quad \Rightarrow \quad a^2-5a=a-5 \quad \Rightarrow \quad a^2-6a+5=0 \quad \Rightarrow \quad (a-5)(a-1)=0 \quad \Rightarrow \quad a=5, a=1$$

34. Use the definition $a^x=y \quad \Leftrightarrow \quad \log_a y=x$ to get $\log_M y=x$

35. $\log_2\left(\frac{1}{4}\right)=x \quad \Rightarrow \quad 2^x=\frac{1}{4} \quad \Rightarrow \quad 2^x=2^{-2} \quad \Rightarrow \quad x=-2$

36. Use the properties $\log(ab)=\log(a)+\log(b)$, $\quad \log\left(\frac{a}{b}\right)=\log(a)-\log(b)$, and $\log x^a=a\log x$

$$2\log_{10}3+4\log_{10}y-6\log_{10}z-8\log_{10}t \quad = \quad \log_{10}3^2+\log_{10}y^4-\log_{10}z^6-\log_{10}t^8 \quad =$$

$$\log_{10}9+\log_{10}y^4-\left(\log_{10}z^6+\log_{10}t^8\right) \quad = \quad \log_{10}\left(9y^4\right)-\log_{10}\left(z^6t^8\right) \quad = \quad \log_{10}\left(\frac{9y^4}{z^6t^8}\right)$$

37. $10^{2x-1}-50=50 \quad \Rightarrow \quad 10^{2x-1}=100 \quad \Rightarrow \quad 10^{2x-1}=10^2 \quad \Rightarrow \quad 2x-1=2 \quad \Rightarrow \quad 2x=3 \quad \Rightarrow \quad x=\frac{3}{2}$

38. $\ln(3x-4)=0 \quad \Rightarrow \quad 3x-4=e^0 \quad \Rightarrow \quad 3x-4=1 \quad \Rightarrow \quad 3x=5 \quad \Rightarrow \quad x=\frac{5}{3}$

39. Solve $85=70+28.6e^{-0.14t}$ for t:

$$85=70+28.6e^{-0.14t} \quad \Rightarrow \quad 15=28.6e^{-0.14t} \quad \Rightarrow \quad \frac{15}{28.6}=e^{-0.14t} \quad \Rightarrow \quad \ln\left(\frac{15}{28.6}\right)=-0.14t \quad \Rightarrow$$

$$t=\frac{1}{-0.14}\ln\left(\frac{15}{28.6}\right)\approx 4.6 \text{ hours}$$

40. Remember $i=\sqrt{-1}$ so $i^2=-1$. Then $7i^{23}-\ 2i\left(3-4i\right)-\left(1+2i\right)^2 \quad = \quad 7i^{23}-\ 2i\left(3-4i\right)-\left(1+4i+4i^2\right)$

$= \quad 7i^{23}-\ 6i+8i^2-1-4i-4i^2 \quad = \quad 7\left(i^2\right)^{11}i^1-\ 6i+8i^2-1-4i-4i^2 \quad =$

$7(-1)^{11}i-\ 6i+8(-1)-1-4i-4(-1) \quad = \quad -7i-\ 6i-8-1-4i+4 \quad = \quad -5-17i$

41. Write $(-1+4i)(a+bi)=2+3i$, then find $a+bi$.

$$a+bi=\frac{2+3i}{-1+4i} \quad = \quad \frac{2+3i}{-1+4i}\cdot\frac{-1-4i}{-1-4i} \quad = \quad \frac{-2-11i-12i^2}{1-16i^2} \quad = \quad \frac{10-11i}{17} \quad = \quad \tfrac{10}{17}-\tfrac{11}{17}i$$

42. Let $z=a+bi$, so $\overline{z}=a-bi$, and we have $(a-bi)-2(a+bi)=6i$.
Simplify the left side to get $-a-3bi=6i \quad \Rightarrow \quad a=0$ and $b=-2$, so $z=-2i$.

43. Substitute n + 1 in place of n in the expression $(-1)^{n+1}\dfrac{n+1}{n^2}$ to get

$$a_{n+2}=(-1)^{(n+1)+1}\frac{(n+1)+1}{(n+1)^2} \quad = \quad (-1)^{n+2}\frac{n+2}{n^2+2n+1}$$

44. You can use the formula for the nth term in an arithmetic sequence: $a_n=a_1+(n-1)(d)$
$26=56+(11-1)(d) \ \Rightarrow \ 26=56+10d \ \Rightarrow \ d=-3$
Since we now have a_1 and d, use the formula to find a_{17}: $a_{17}=56+(16)(-3) \ = \ 8$

45. Use the formula for the nth term in a geometric sequence: $a_n=a_1\cdot r^{n-1}$, with $a_1=2$ and $a_3=7$:
$7=2\cdot r^{3-1} \ \Rightarrow \ \tfrac{7}{2}=r^2 \ \Rightarrow \ \pm\sqrt{\tfrac{7}{2}}=r$
But since we are told that all the terms in the sequence are positive, this rules out the possibility of a negative value for r, so $r=\sqrt{\tfrac{7}{2}}$.

46. Use the formula for the nth term in a geometric sequence: $a_n=a_1\cdot r^{n-1}$ to get a system of two equations:

$$\begin{cases} 3=a_1\cdot r^{2-1} \ \Rightarrow \ 3=a_1r, \text{ and} \\ 9=a_1\cdot r^{4-1} \ \Rightarrow \ 9=a_1r^3 \end{cases}$$

We can substitute 3 in place of a_1r in the second equation to get $9=3r^2 \ \Rightarrow \ r=\pm\sqrt{3}$. Since all of the choices given are positive values for the seventh term of the sequence, r must be positive, so we use $\sqrt{3}$.
Now we can use $r=\sqrt{3}$ to find a_1, then use the formula for the nth term to find a_7:

$3=a_1\sqrt{3} \ \Rightarrow \ a_1=\frac{3}{\sqrt{3}}$, and $a_7=\frac{3}{\sqrt{3}}\left(\sqrt{3}\right)^{7-1} \ = \ 3\left(\sqrt{3}\right)^5=27\sqrt{3}$

47. $a_1 = 2+3i$

$a_2 = i\cdot a_1 \;=\; i(2+3i) \;=\; 2i+3i^2 \;=-3+2i$

$a_3 = i\cdot a_2 \;=\; i(-3+2i) \;=\; -3i+2i^2 \;=-2-3i$

$a_4 = i\cdot a_3 \;=\; i(-2-3i) \;=\; -2i-3i^2 \;=3-2i$

$a_5 = i\cdot a_4 \;=\; i(3-2i) \;=\; 3i-2i^2 \;=2+3i$

48. $\sum_{n=1}^{23}(6n-3) = (6\cdot1-3)+(6\cdot2-3)+(6\cdot3-3)+...+(6\cdot23-3) \;=\; 3+9+15+\ldots+135.$

We can see that this is a sum of the first 23 terms in an arithmetic sequence with common difference $d=6$, so use the formula for the sum of the first *n* terms in an arithmetic sequence:

$S_{23} = \frac{23}{2}\left[a_1+a_{23}\right] \;=\; 11.5(3+135) \;=1587$

49. We want to find the sum of the first 10 terms in an arithmetic sequence, with first term $a_1 = 60$ and common difference $d=-2$. Use the formula for the *n*th term in an arithmetic sequence to find the number of cans in the 10th row: $a_{10} = 60+(10-1)(-2) \;=\; 42$ cans. Then use the formula for the sum of the first *n* terms in an arithmetic sequence: $S_{10} = \frac{10}{2}\left[a_1+a_{10}\right] \;=\; 5(60+42) \;=\; 510$ cans

50. $\frac{9!}{3!\,6!} \;=\; \frac{9\cdot8\cdot7\cdot6!}{3\cdot2\cdot1\cdot6!} \;=\; \frac{9\cdot8\cdot7}{3\cdot2\cdot1} \;=\; 84$

51. Multiply each entry in *B* by 5, then add the corresponding entries in *A* and 5*B*. The entry in the first row first column is $8+{}^{-}15=-7$.

52. Multiply the two matrices on the left side of the equation to get $\begin{bmatrix}2k & 8\\ 5k & 17\end{bmatrix} = \begin{bmatrix}6 & 8\\ 15 & 17\end{bmatrix}$. The entries in the two matrices must all be the same, so we have $2k=6$ and $5k=15$, which imply $k=3$.

53. Apply the definition $\begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad-bc$ to the left side of the equation to get $\begin{vmatrix}k & 1\\ 4 & 2\end{vmatrix} = 10 \;\Rightarrow 2k-4=10$, then solve for k to get $k=7$.

54. One method is to rewrite the system as: $\begin{cases}4x & -5y & & =11\\ 2x & & +z & =7\\ & 2y & +z & =1\end{cases}$

Then multiply the 2nd equation by -2 and add the result to the 1st equation to eliminate x, which gives $-5y-2z=-3$. Using this result with the original 3rd equation, we have a system of two equations with two variables: $\begin{cases}-5y & -2z & =-3\\ 2y & +\;z & =\;1\end{cases}$

We want eliminate *y* now, so multiply the top equation by 2 and the bottom equation by 5. When we add them together and solve for *z*, we get *z* = –1.

55. First, write the terms in the same order to get $\begin{aligned} 4x - 5y &= 11 \\ -2x + y &= 7 \end{aligned}$

Then, write a matrix where each row gives the coefficients of an equation. One type of notation for an augmented matrix uses a dashed line to indicate the augmentation of the coefficient matrix with the matrix of constant terms: $\left[\begin{array}{rr:r} 4 & -5 & 11 \\ -2 & 1 & 7 \end{array}\right]$

56. One way to proceed is multiply both sides of the inequality by -3, but this requires us to switch the direction of the inequality symbol: $x^2 - 4 \le -3x$. Then one method to finish the solution is to write the inequality with zero on one side and examine the sign changes of the other side: $x^2 + 3x - 4 \le 0 \quad \Rightarrow \quad (x+4)(x-1) \le 0$. The product on the left side will be equal to 0 when either factor is 0, which occurs when $x = -4$ or when $x = 1$. The product on the left side will be less than 0 when one of the factors is negative and the other is positive. Since for all values of *x*, the factor $x + 4$ is larger than the factor $x - 1$, the positive factor with be $x + 4$ and the negative factor will be $x - 1$. This means $x + 4 > 0$ and $x - 1 < 0$, which implies $x > -4$ and $x < 1$, which means $-4 < x < 1$. So $(x+4)(x-1) \le 0$ is true when $-4 \le x \le 1$, or when *x* is in the interval $[-4,1]$.

57. The only graphs that pass the vertical line test are I and II.

58. Distribute the -3 through the components of **b** and then add the corresponding components of **a** and $-3\mathbf{b}$:
$\mathbf{a} - 3\mathbf{b} = \langle 1,3 \rangle - 3\langle 2,-5 \rangle \quad = \quad \langle 1,3 \rangle + \langle -6,15 \rangle \quad = \quad \langle -5,18 \rangle$.

59. $\mathbf{a} \cdot \mathbf{b} = \langle 1,3 \rangle \cdot \langle 2,-5 \rangle \quad = \quad (1)(2) + (3)(-5) \quad = \quad 2 - 15 \quad = \quad -13$

60. When the point $(2,-4)$ is reflected across the *x*-axis, we get the point $(2,4)$. Then, if we translate the point $(2,4)$ by the vector $\langle 3,-1 \rangle$, we move right 3 and down 1 to end up at $(5,3)$.

61. A function from *A* to *B* must map each element of *A* to one element of *B* (but it is not necessary to use all of the elements of *B*). There are many possible functions from *A* to *B*, so the only way to answer this question is to find which of the choices works. We can try evaluating each function using the elements of *A* to determine whether or not we get values that are in *B*.

a. $f(1) = -3(1) + 23 = 20$, which is in *B*
$f(3) = -3(3) + 23 = 14$, which is NOT in *B*, so this choice is incorrect.

b. $f(1) = 6(1) + 2 = 8$, which is in *B*
$f(3) = 6(3) + 2 = 20$, which is in *B*
$f(7) = 6(7) + 2 = 44$, which is NOT in *B*, so this choice is incorrect.

c. $f(1) = 1^2 - 1 + 2 = 2$, which is in *B*
$f(3) = 3^2 - 3 + 2 = 8$, which is in *B*
$f(7) = 7^2 - 7 + 2 = 44$, which is NOT in *B*, so this choice is incorrect. Choice d had better work.

d. $f(1) = 3(1) - 1 = 2$, which is in *B*
$f(3) = 3(3) - 1 = 8$, which is in *B*
$f(7) = 3(7) - 1 = 20$, which is in *B*, ***so this choice is correct.***

62. As in question 61, there are many possible equations that could relate the variables. We can try the equations to see which one works for each pair of data values.

a. $y = x - 14$ works when $x = 20$, but not for $x = 40$ since $40 - 14 = 26 \neq 16$. So this choice is incorrect.

b. $y = 10\log_2\left(\frac{x}{10}\right) - 4$ works for each pair of data values:

When $x = 20$ $y = 10\log_2\left(\frac{20}{10}\right) - 4 \;=\; 10\log_2 2 - 4 \;=\; 10(1) - 4 \;=\; 6$, and

when $x = 40$ $y = 10\log_2\left(\frac{40}{10}\right) - 4 \;=\; 10\log_2 4 - 4 \;=\; 10(2) - 4 \;=\; 16$, and

when $x = 80$ $y = 10\log_2\left(\frac{80}{10}\right) - 4 \;=\; 10\log_2 8 - 4 \;=\; 10(3) - 4 \;=\; 26$. ***So this choice is correct.***

(Choices c and d do not work for any of the pairs of data values.)