Precalculus Final Exam Review

Revised Fall 2015

1. $f(x)$ is a function that generates the ordered pairs (0,0), (1,7) and (2,-3).
   a. If $f(x)$ is an odd function, what are the coordinates of two other points found on the graph of $f(x)$?
   b. If $f(x)$ is an even function, what are the coordinates of two other points found on the graph of $f(x)$?

2. Using Algebraic techniques, determine whether $f(x) = x^3 + 2x - 3$ is even, odd, or neither.

3. Given: $f(x) = e^x$, $g(x) = 2x^2 - 5x - 3$, and $k(x) = x^7$,
   Find:

   a. $g(f(x))$ b. $(f \cdot g)(x)$ c. $\dfrac{g(x+h) - g(x)}{h}$

   d. $f(g(1))$ e. the average rate of change of $g(x)$ between $x = 0$ and $x = 3$.

   f. $\dfrac{k(x+h) - k(x)}{h}$ g. $(k \circ f)(x)$

4. Let $f(x) = \begin{cases} 2x - 10 & 0 \le x < 50 \\ x + 30 & 50 \le x \le 150 \end{cases}$
   a. State the domain of $f(x)$. Use any appropriate notation.
   b. Find $f(83)$.
   c. Graph this function. Clearly show your axes scales and label any important points.

5. The function $D(p) = 1200 - 200p$ gives the weekly demand for video rentals at Joe's Videolog when Joe charges p dollars to rent a video.
   a. What is the demand when Joe charges \$4.50 to rent a video?
   b. Find $D^{-1}(50)$
   c. Interpret $D^{-1}(50)$ in context of this problem.

6. The graph of function $y = 2x^2$ is to be shifted horizontally 3 units right and vertically 5 units up. Write the function definition that represents this transformation.

7. Let $C = f(x)$ be the circumference of a circle with radius x cm.
   a. Explain the meaning of $C = f(x+2)$.
   b. Explain the meaning of $C = f(x) + 2$.

8. The following table gives function values for $f(x)$ and $g(x)$ for $x = 1,2,3,4,5$. Complete the table for $m(x) = \dfrac{g(x)}{f(x)}$ and $n(x) = f(g(x))$. Place an X in any boxes where the value cannot be determined.

x	1	2	3	4	5
$f(x)$	4	3	5	2	3
$g(x)$	3	4	0	2	1
$m(x)$					
$n(x)$					

9. For the following functions, find all the zeros and describe the long-run behavior:

   a. $f(x) = 3x^3 - 8x^2 - 20x + 16$

   b. $f(x) = 12x^4 + 41x^3 - 108x^2 - 59x + 30$

   c. $f(x) = 2x^3 - 23x^2 + 76x - 69$

   d. $f(x) = 5x^4 - 19x^3 + 6x^2 + 22x + 4$

   e. $f(x) = -x^2 + 2x - 5$

   f. $f(x) = 3x^3 - 11x^2 + 17x + 7$

10. Determine whether each table could represent a linear, exponential or periodic function and then find a possible formula for each.

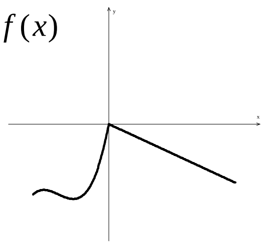
x	$f(x)$
0	1.2
1	2.4
2	4.8
3	9.6
4	19.2

x	$g(x)$
0	1.2
1	2.4
2	3.6
3	4.8
4	6.0

x	$h(x)$
0	1.2
1	2.4
2	1.2
3	0
4	1.2

11. Using the figure shown below, match the formulas (i) - (vi) with a graph from (a) - (f).

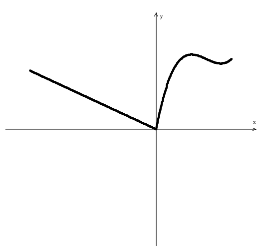
$f(x)$



(i) $y = f(-x)$ (ii) $y = -f(x)$ (iii) $y = f(-x) + 3$

(iv) $y = -f(x-1)$ (v) $y = -f(-x)$ (vi) $y = -2 - f(x)$

a.



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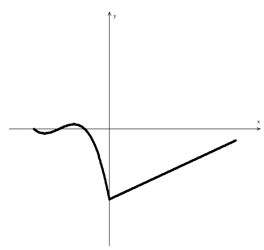

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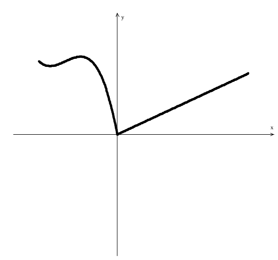
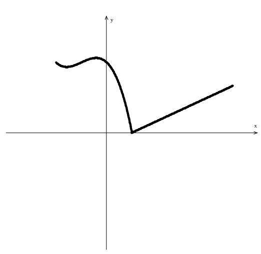
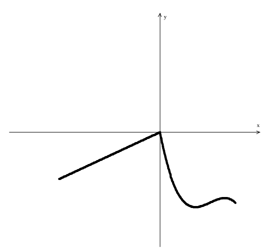


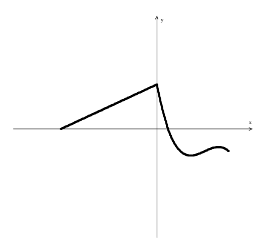
d.



e.



f.



12. Sketch a graph of a function $y = f(x)$ with <u>all</u> of the following features...

   o $f(0) = -3$

   o $f(-4) = f(2) = 0$

   o f is decreasing for $x < 0$

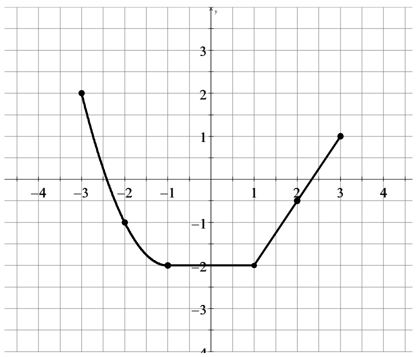
   o f is increasing for $x > 0$

   o as $x \to \infty$, $f(x) \to 2$

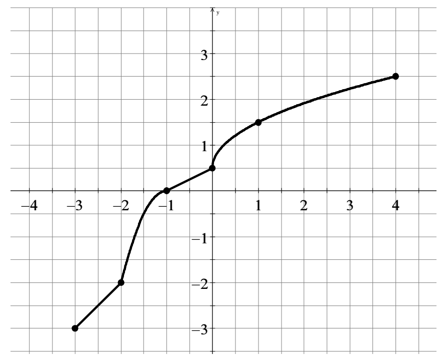
   o as $x \to -\infty$, $f(x) \to \infty$

13. Use the graphs of *f* and *g* below to find $g^{-1}(0)$, $g(f(3))$, $(fg)(-3)$, $\left(\frac{g}{f}\right)(-1)$, $(f+g)(-2)$, $(g-f)(-3)$.

$f(x)$

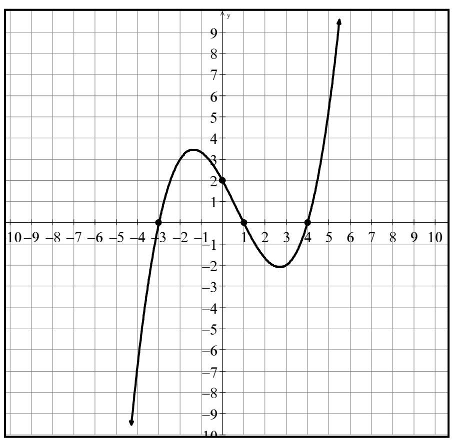


$g(x)$

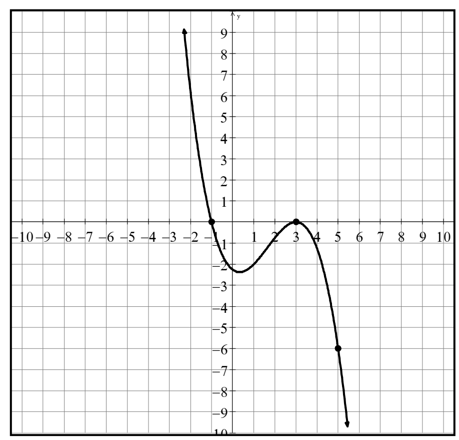


14. Write a possible formula for the polynomials graphed below. Leave your answers in factored form.

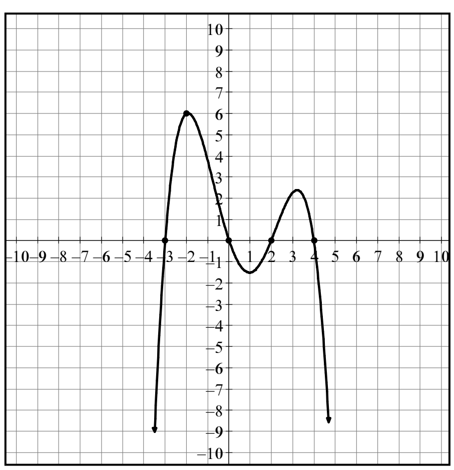
a.



b.



c.



d. Describe the concavity of each function at $x = 3$.

15. State the domain and range of the following.

a. $y = e^x$ b. $y = 3\ln x$ c. $y = x^2 - 9$ d. $y = \tan x$

e. $y = 3x$

16. Find the domain, x-intercepts (if any), y-intercepts (if any), asymptotes (if any), and describe the long-run behavior.

a. $P(x) = \dfrac{-2x+1}{5x-4}$ b. $Q(x) = \dfrac{2x+1}{x^2-1}$ c. $R(x) = \dfrac{3x^3}{x^2-9}$

17. Find the domain, range and any asymptotes for each of the following.

a. $y = \log_5(x-3)$ b. $y = 3\ln x - 2$ c. $y = 2e^x + 1$

d. $y = -2^{x-3}$

18. Solve. Give your solutions in exact form.

a. $\log x + \log(2x+1) = 1$
b. $\ln(3x+5) = 3$
c. $6e^{2x+1} + 4 = 16$
d. $10^{x^2+3} = 10{,}000$

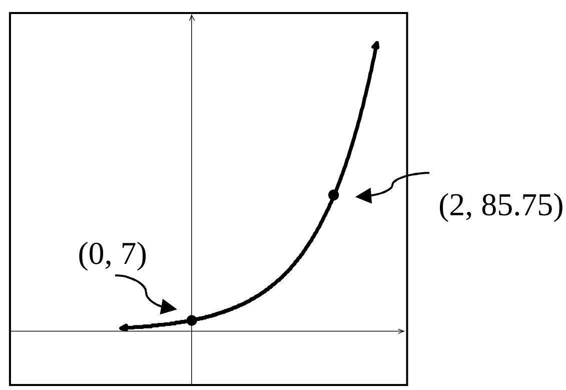
19. The population of a city has an **annual** growth rate of 1.2% per year. If the initial population of the city is 152,000:

a. Find a formula for $P(t)$, the population in year t.
b. What will the population be in 10 years?
c. How many years will it take the population to double? Round to the nearest year.

20. A bank account earns a **continuous** interest rate of 6%. If \$5,000 is deposited into the account:

a. Find a formula for $B(t)$, the balance in the account in t years.
b. When will the balance reach \$50,000? Give an exact answer and then round to the nearest tenth of a year.

21. The graph at the right represents an exponential function of the form $f(t) = ab^t$. Find a and b exactly.



22. Complete the problems which involve series.

a. Evaluate the sum $\sum_{k=1}^{50}(-3k-5)$

b. Evaluate the sum $\sum_{n=0}^{10} 3(2)^n$

c. Express the given series using compressed, $\sum$, notation $3 + \frac{4}{3} + \frac{-1}{3} + (-2) + \frac{-11}{3}$

d. Express the given series using compressed, $\sum$, notation $3 + 51 + 867 + 14739$

23. Use the Binomial Theorem to expand $(2x+3y^2)^4$. Simplify completely.

24. a. Convert $210°$ to radians. b. Convert $-\frac{3\pi}{2}$ to degrees.

25. Find the exact value of the six trigonometric functions of θ if $\cos\theta = -\frac{3}{5}$ and θ is in Quadrant III.

26. Find the exact value of:

a. $\tan\left(\sin^{-1}\left(\frac{1}{5}\right)\right)$ b. $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$ c. $\sec\left(\tan^{-1}\left(\frac{1}{x}\right)\right)$

27. Solve each triangle. Round answers to one decimal place.

a. A = 35°, C = 90°, b = 5 b. A = 25°, b = 2, c = 5

c. A = 10°, a = 3, b = 4

28. From a point on level ground 135 feet from the base of a tower, the angle of elevation of the top of the tower is 57.3°. Approximate the height of the tower rounded to the nearest foot.

29. The angle at one corner of a triangular plot of ground is 73.7° and the sides that meet at this corner are 175 feet and 150 feet long. Approximate the length of the third side rounded to the nearest foot.

30. For each function, identify the midline, amplitude, period, horizontal shift, and asymptotes, when appropriate.

a. $y = 3\cos\left(\frac{1}{2}x\right) - 1$ b. $y = 4\cos\,(3x - \pi)$ c. y = -2 sin x d. y = tan (2x)

31. Find solutions for each equation.

a. On the interval [0, 2π) give exact answers.

b. On the interval [0, 2π) give approximate answers rounded to 4 decimal places.

i. $2\sin\theta - 3\sin\theta\cos\theta = 0$

ii. $2\sin^2 x + \sin x - 1 = 0$

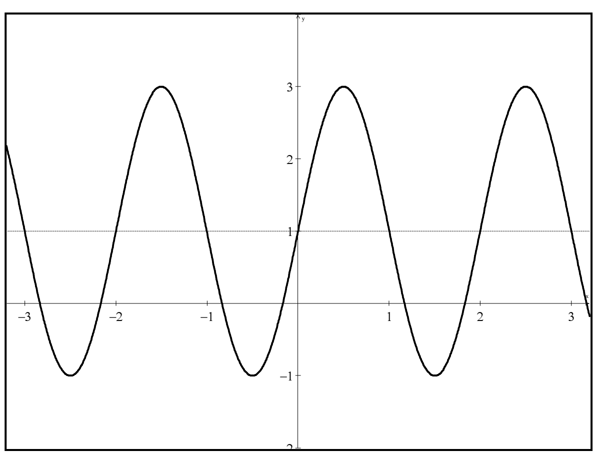
iii. $\sin(2x) - \sin x - 2\cos x = -1$

iv. $4\tan x\,\sin x = -\sin x$

c. Use your answers from part (a) to find all real solutions.

32. A person's blood pressure, P, (in millimeters of mercury) is given by $P = 100 - 20\cos\left(\frac{8\pi}{3}t\right)$, where t is time in seconds. State the period, the midline and the amplitude and explain the practical significance of these quantities.

33. Assume the graph below shows a portion of a **sine** curve. Find the amplitude, the period, the equation of the midline, and any horizontal shift. Next, write a function definition for the curve. Finally, state the domain and range for the function.



34. Verify each identity:

a. $(\sin x + \cos x)^2 = 1 + \sin(2x)$

b. $4\sin^2 x + 2\cos^2 x = 4 - 2\cos^2 x$

c. $\dfrac{1}{1-\cos\theta} - \dfrac{1}{1+\cos\theta} = 2\cot\theta\csc\theta$

d. $1 - \dfrac{\cos^2 x}{1+\sin x} = \sin x$

e. $(\cot x + \tan x)^2 = \sec^2 x + \csc^2 x$

35. Find the magnitude and direction of vector $\vec{v} = 5\vec{i} + 14\vec{j}$. Round answers to one decimal place.

36. A cyclist rides 6 mph due north as the wind blows him 4 mph west.

a. Draw a sketch of this situation using one horizontal vector and one vertical vector.

b. On your sketch, draw the resultant vector which show the cyclist's total displacement. Label this vector as $\vec{v}$.

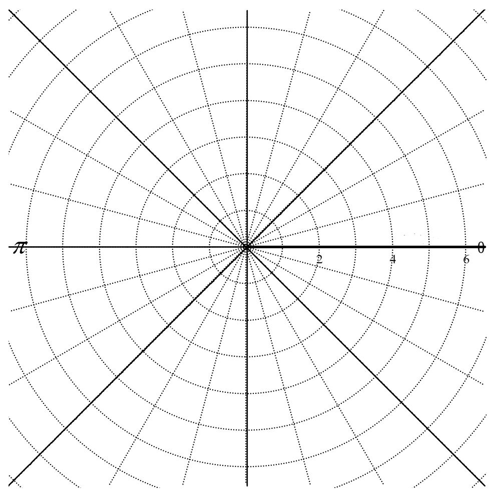
c. Express the resultant vector, $\vec{v}$, from part (b) in component form. For vector $\vec{v}$ find the exact magnitude and direction correct to 1 decimal place.

37. For the point $P\left(5, \dfrac{4\pi}{3}\right)$:

a. Plot the point on the graph paper at right.

b. Find another polar representation of the same point for $r = -5$.

c. Convert the point $P\left(5, \dfrac{4\pi}{3}\right)$ to exact Cartesian coordinates.



38. Sketch the polar curves. For each curve, give an interval of θ for which the curve is traced exactly once. State the exact values of three distinct points in polar coordinates for each curve.

a. $r = 4\cos 2\theta$

b. $r = 2\sin\theta$

c. $r = 2 - 2\sin\theta$

Answer Key - Precalculus Final Exam Review

1a. (-1, -7) and (-2, 3)   1b. (-1, 7) and (-2, -3)   2. neither

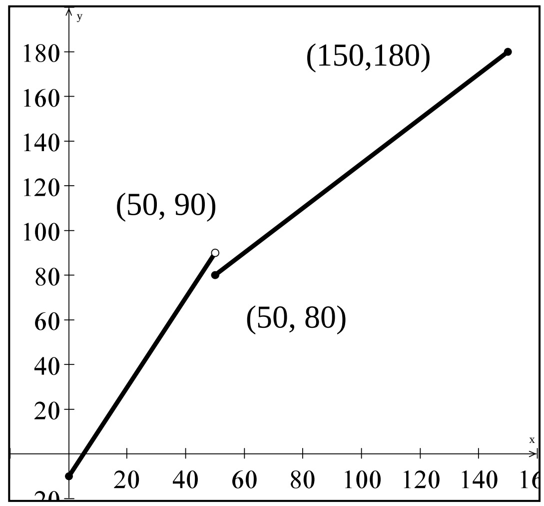
3a. $g(f(x)) = 2e^{2x} - 5e^{x} - 3$ 3b. $(f \cdot g)(x) = e^{x}\left(2x^2 - 5x - 3\right)$

3c. $\dfrac{g(x+h) - g(x)}{h} = 4x + 2h - 5$ 3d. $f\left(g(1)\right) = e^{-6}$ 3e. 1

3f. $7x^6 + 21x^5h + 35x^4h^2 + 35x^3h^3 + 21x^2h^4 + 7xh^5 + h^6$

3g. e^{7x}

4a. [0, 150]   4b. f(83) = 113   4c.



5a. D(4.50) = 300 video rentals per week
5b. p = \$5.75
5c. When the demand is 50 rentals weekly, the price is \$5.75.

6. $f(x) = 2(x-3)^2 + 5$

7a. The circumference of a circle when the radius is increased by 2 cm.
7b. The circumference of a circle is increased by 2 cm.

8.

$m(x)$	3/4	4/3	0	1	1/3
$n(x)$	5	2	X	3	4

9a. $\{-2, 2/3, 4\}$ as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to -\infty$

9b. $\{-5, -3/4, 1/3, 2\}$ as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to \infty$

9c. $\left\{3/2, 5+\sqrt{2}, 5-\sqrt{2}\right\}$ as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to -\infty$

9d. $\left\{-1/5, 2, 1+\sqrt{3}, 1-\sqrt{3}\right\}$ as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to \infty$

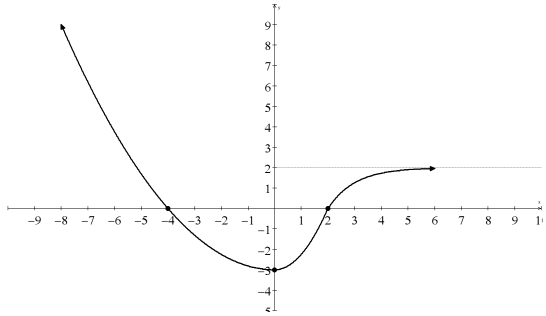
9e. $\{1+2i, 1-2i\}$ as $x \to \infty, y \to -\infty$ and as $x \to -\infty, y \to -\infty$

9f. $\left\{-1/3, 2+i\sqrt{3}, 2-i\sqrt{3}\right\}$ as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to -\infty$

10. $f(x) = 1.2(2)^x$ exponential; $g(x) = 1.2x + 1.2$ linear; $h(x) = 1.2\sin\left(\dfrac{\pi}{2}x\right) + 1.2$ periodic

11a. v   11b. vi   11c. ii   11d. iv   11e. i   11f. iii

12. Answers vary



13. $g^{-1}(0)=-1$; $g(f(3))=1.5$; $(fg)(-3)=-6$; $\left(\frac{g}{f}\right)(-1)=0$; $(f+g)(-2)=-3$; $(g-f)(-3)=-5$

14a. $y=\frac{1}{6}(x-1)(x+3)(x-4)$

14b. $y=-\frac{1}{4}(x+1)(x-3)^2$

14c. $y=-\frac{1}{8}x(x+3)(x-2)(x-4)$

14d. a is concave up at x = 3, b is concave down at x = 3, c is concave down at x = 3

15a. Domain (-∞, ∞)
Range (0, ∞)

15b. Domain (0, ∞)
Range (-∞, ∞)

15c. Domain (-∞, ∞)

Range [-9, ∞)

15d. Domain $\left\{x \mid x \neq \frac{k\pi}{2}\right\}$, k an odd integer

Range (-∞, ∞)

15e. Domain (-∞, ∞)
Range (-∞, ∞)

16a. Domain: $\left\{x \mid x \neq \frac{4}{5}\right\}$, x-int: (0.5, 0), y-int: (0, -0.25), VA: $x=\frac{4}{5}$,
HA: $y=-\frac{2}{5}$, as $x \to \pm\infty$, $y \to -\frac{2}{5}$

16b. Domain: $\{x \mid x \neq \pm 1\}$, x-int: (-0.5, 0), y-int: (0, -1), VA: x = 1, x = -1,
HA: y = 0, as $x \to \pm\infty$, $y \to 0$

16c. Domain: $\{x \mid x \neq \pm 3\}$, x-int: (0, 0), y-int: (0, 0), VA: x = 3, x = -3,
as $x \to \infty$, $y \to \infty$ and as $x \to -\infty$, $y \to -\infty$

17a. D = (3, ∞)
R = (-∞, ∞)
VA : x = 3

17b. D = (0, ∞)
R = (-∞, ∞)
VA : x = 0

17c. D = (-∞, ∞)
R = (1, ∞)
HA : y = 1

17d. D = (-∞, ∞)
R = (-∞, 0)
HA : y = 0

18a. x = 2, (x = -5/2 is extraneous)

18b. $x=\frac{1}{3}(e^3-5)$

18c. $x=\frac{1}{2}(-1+\ln 2)$

18d. $x=\pm 1$

19a. $P(t)=152000(1.012)^t$

19b. P(10)=171,257 people

19c. 58 years

20a. $B(t)=5000e^{0.06t}$

20b. $t=\frac{\ln 10}{0.06}\approx 38.4$ years

21. a = 7, b = 3.5

22a. $S_{50} = -4075$ 22b. $3 + S_{10} = 6{,}141$ 22c. $\sum_{n=0}^{4}\left(3 - \frac{5}{3}n\right)$ or $\sum_{n=1}^{5}\left(-\frac{5}{3}n + \frac{14}{3}\right)$

22d. $\sum_{n=0}^{3} 3(17)^n$ or $\sum_{n=1}^{4} 3(17)^{n-1}$ There are many possibilities for 22c and 22d.

23. $16x^4 + 96x^3y^2 + 216x^2y^4 + 216xy^6 + 81y^8$

24a. $\frac{7\pi}{6}$ 24b. -270°

25. $\sin\theta = -\frac{4}{5}, \cos\theta = -\frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = -\frac{5}{4}, \sec\theta = -\frac{5}{3}, \cot\theta = \frac{3}{4}$

26a. $\frac{1}{2\sqrt{6}}$ 26b. $\frac{\sqrt{3}}{2}$ 26c. $\frac{\sqrt{1+x^2}}{x}$

27a.	c = 6.1	a = 3.5	B = 55°
27b.	a = 3.3	B = 14.9°	C = 140.1°
27c.	B = 13.4°	C = 156.6°	c = 6.9
	or		
	B = 166.6°	C = 3.4°	c = 1.0

28. 210 ft 29. 196 ft

30a. midline: y = -1	Amp = 3	per = 4π	horizontal shift 0
30b. midline: y = 0	Amp = 4	per = $\frac{2\pi}{3}$	horizontal shift $\frac{\pi}{3}$ right
30c. midline: y = 0	Amp = 2	per = 2π	horizontal shift 0

30d. no midline per = $\frac{\pi}{2}$ VA at these x-values $\frac{\pi}{4} + \frac{k\pi}{2}$ where k is an integer

31a. i. $\left\{0, \pi, \cos^{-1}(2/3), 2\pi - \cos^{-1}(2/3)\right\}$ ii. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

iii. $\left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}\right\}$ iv. $\left\{0, \pi, \pi + \tan^{-1}(-1/4), 2\pi + \tan^{-1}(-1/4)\right\}$

31b. i. $\{0, 3.1416, 0.8411, 5.4421\}$ ii. $\{0.5236, 2.6180, 4.7124\}$

iii. $\{1.0472, 1.5708, 5.2360\}$ iv. $\{0, 3.1416, 2.8966, 6.0382\}$

31c. In the following solutions, k represents any integer.

i. $\left\{k\pi, \cos^{-1}(2/3) + 2k\pi, 2k\pi - \cos^{-1}(2/3)\right\}$ ii. $\left\{\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{3\pi}{2} + 2k\pi\right\}$

iii. $\left\{\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi, \frac{5\pi}{3} + 2k\pi\right\}$ iv. $\left\{k\pi, \tan^{-1}(-1/4) + k\pi\right\}$

32. period = 0.75 sec
midline: P = 100
amplitude = 20 mm of mercury
Blood pressure fluctuates between a low of 80 and a high of 120 in 0.75 seconds.

33. A = 2 per = 2 midline: y = 1 horizontal shift: none
$f(t) = 2\sin(\pi t) + 1$ Domain: $(-\infty, \infty)$ Range: [-1, 3]

34. proofs - suggested first steps

a. Square the left hand side then use a Pythagorean identity.

b. Use a Pythagorean identity to replace $\sin^2 x$ with $1-\cos^2 x$ on the left-hand side.

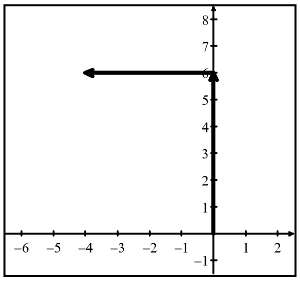
c. Find a common denominator on the left-hand side to combine the two fractions into one fraction.

d. Use a Pythagorean identity to replace $\cos^2 x$ with $1-\sin^2 x$ on the left-hand side.

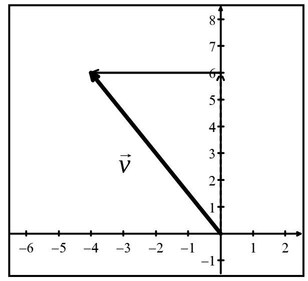
e. Square the left-hand side.

35. $|\vec{v}| = \|\vec{v}\| = 14.9$ $\theta = 70.3^\circ$

36a.

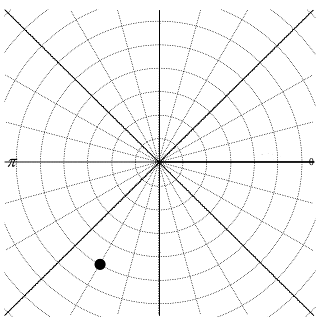


36b.



36c. $\vec{v} = -4\vec{i} + 6\vec{j}$, $|\vec{v}| = \|\vec{v}\| = 2\sqrt{13}$, $\theta = 33.7^\circ$ west of north

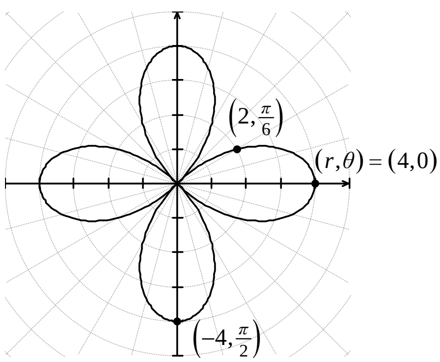
37a.



37b. $\left(-5, \frac{\pi}{3}\right)$

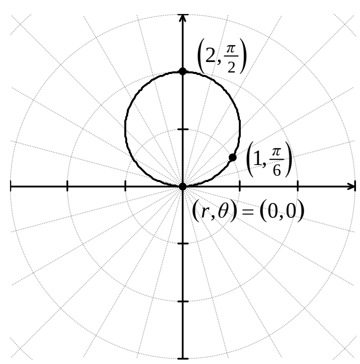
37c. $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$

38a.



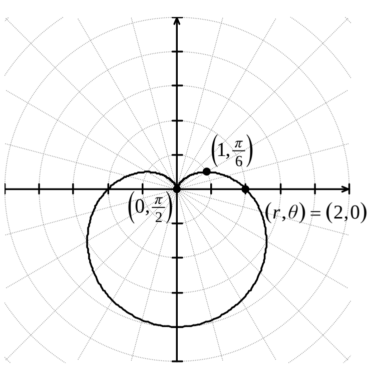
$0 \le \theta < 2\pi$ traces once

38b.



$0 \le \theta < \pi$ traces once

38c.



$0 \le \theta < 2\pi$ traces once