JOHNSON COUNTY COMMUNITY COLLEGE
Calculus III Final Exam Review

This final review will be a useful ***starting point*** as you study for your final exam. You should also study your tests and homework from this semester. **There are concepts on the final exam that are *not* covered on this review.**

1. Let $\mathbf{r}'(t) = \cos 2t\,\mathbf{i} - 2\sin t\,\mathbf{j} + \dfrac{1}{1+t^2}\mathbf{k}$ with $\mathbf{r}(0) = \langle 3, -2, 1\rangle$. Find $\mathbf{r}(t)$.

2. Find the unit tangent vector for $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\,\mathbf{k}$ at $t = \frac{\pi}{4}$.

3. For $f(x,y) = xe^{x^2 y}$, find f_x and f_y at the point $(1, \ln 2)$.

4. Find the directional derivative of $f(x,y) = 4 - x^2 - \dfrac{y^2}{4}$ at (1,2) in the direction of $\mathbf{v} = \langle 3, 4\rangle$.

5. What is the direction of maximum increase of $f(x,y,z) = x^2 + y^2 - 4z$ from the point $(2, -1, 1)$?

6. Find an equation of the tangent plane to the hyperboloid $z^2 = 12 + 2x^2 + 2y^2$ at the point $(1, -1, 4)$.

7. Find the relative extrema and/or saddle points of the function $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$.

8. Find the relative extrema and/or saddle points of the function $f(x,y) = x^3 - 3xy + y^3$.

9. Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint that (x,y,z) is restricted to the plane $2x - 3y + 4z = 49$.

10. Evaluate $\displaystyle\int_0^2 \int_0^x \int_0^{x+y} e^x (y + 2z)\,dz\,dy\,dx$.

11. Find the volume of the solid region Q cut from the sphere $x^2 + y^2 + z^2 = 4$ by the cylinder $r = 2\sin\theta$.

12. Find the volume of the solid region Q bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and bounded above by the sphere $x^2 + y^2 + z^2 = 9$.

13. Find the work done by the force field $\mathbf{F}(x, y, z) = -\frac{1}{2}x\,\mathbf{i} - \frac{1}{2}y\,\mathbf{j} + \frac{1}{4}\mathbf{k}$ on a particle as it moves along the helix determined by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$ from the point $(1,0,0)$ to $(-1,0,3\pi)$.

14. Let C be the circle of radius 3 determined by $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$, where $0 \le t \le 2\pi$ and evaluate the line integral $\int_C y^3\,dx + \left(x^3 + 3xy^2\right)dy$.

15. While subject to the force $\mathbf{F}(x, y) = y^3\,\mathbf{i} + \left(x^3 + 3xy^2\right)\mathbf{j}$, a particle travels once counterclockwise around the circle of radius 3 centered at the origin. Find the work done by **F** on this particle.

16. Evaluate the surface integral $\iint_S \left(y^2 + 2yz\right)d\sigma$, where S is the first octant portion of the plane $2x + y + 2z = 6$.

17. Find the flux of $\mathbf{F}(x, y, z) = \left\langle y^3, -xy, xz\right\rangle$ across S, the first octant portion of the plane $x + y + z = 1$, where **n** is directed away from the origin.

18. Find the outward flux of $\mathbf{F}(x, y, z) = y^3\,\mathbf{i} - xy\,\mathbf{j} + xz\,\mathbf{k}$ across S, the closed tetrahedron formed by the planes $x = 0$, $y = 0$, $z = 0$, and the first octant portion of the plane $x + y + z = 1$.

ANSWERS

1. $\mathbf{r}(t) = \left(\frac{1}{2}\sin 2t + 3\right)\mathbf{i} + \left(2\cos t - 4\right)\mathbf{j} + \left(\arctan t + 1\right)\mathbf{k}$

2. $\mathbf{T}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{\sqrt{5}}\mathbf{i} + \frac{\sqrt{2}}{\sqrt{5}}\mathbf{j} + \frac{1}{\sqrt{5}}\mathbf{k}$

3. $f_x(1, \ln 2) = 2 + 4\ln 2$; $f_y(1, \ln 2) = 2$

4. –2

5. In the direction of $\nabla f = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ (or $\mathbf{u} = \frac{\nabla f}{|\nabla f|} = \frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$)

6. $x - y - 2z + 6 = 0$

7. Relative Minimum is –4 when $(x, y) = (-1, 1)$

8. Saddle point (0,0,0); relative minimum is –1 when $(x, y) = (1, 1)$

9. Minimum is 147, when $(x, y, z) = (3, -9, 4)$

10. $\frac{19}{3}\left(e^2 + 3\right)$

11. $\frac{16}{9}(3\pi - 4)$

12. $9\pi\left(2 - \sqrt{2}\right)$

13. $\frac{3\pi}{4}$

14. $\frac{243\pi}{4}$

15. $\frac{243\pi}{4}$

16. $\frac{243}{2}$

17. $\frac{1}{20}$

18. 0